Algebra 1– UNIT 5
Quadratic Functions and Modeling

Critical Area: In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows x+1 = 0 to have a solution. Formal work with complex numbers comes in Algebra II. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

CLUSTER HEADINGS	COMMON CORE STATE STANDARDS
(s)Use properties of rational and irrational numbers. *Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.*	**Number and Quantity - The Real Number System** N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
(m)Interpret functions that arise in applications in terms of a context. *Focus on quadratic functions; compare with linear and exponential functions studied in Unit 2.*	**Functions - Interpreting Functions** F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for he function.*★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
(m)Analyze functions using different representations. *For F.IF.7b, compare and contrast absolute value, step and piecewise defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining*	**Functions - Interpreting Functions** F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and

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piecewise defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Unit 2 on exponential functions with integer exponents. For F.IF.9, focus on expanding the types of functions considered to include, linear, exponential, and quadratic. *Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.*	minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$ *and classify them as representing exponential growth or decay.* F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
(m)Build a function that models a relationship between two quantities. *Focus on situations that exhibit a quadratic relationship.*	**Functions - Building Functions** F.BF.1 Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
(*s*)Build new functions from existing functions. *For F.BF.3, focus on quadratic functions, and consider including absolute value functions. For F.BF.4a, focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as f(x) = x2, x>0.*	**Functions - Building Functions** F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* F.BF.4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) = 2 x3 or f(x) = (x+1)/(x-1) for x ≠ 1.*
(*s*)Construct and compare linear, quadratic, and exponential models and	**Functions – Linear, Quadratic, and Exponential Model**

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solve problems. *Compare linear and exponential growth to quadratic growth.*	F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
(*s*)Interpret expressions for functions in terms of the situation they model.	**Functions – Linear, Quadratic, and Exponential Model** F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.★ [Linear and exponential of form $f(x)=b^x+k$.] F.LE.6. Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity. ★ CA
MATHEMATICAL PRACTICES	
1. Make sense of problems and persevere in solving them. **2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the reasoning of others.** **4. Model with mathematics.** 5. Use appropriate tools strategically. **6. Attend to precision.** **7. Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	Emphasize Mathematical Practices 1, 2, 3, 4, 6, and 7 in this unit.
LEARNING PROGRESSIONS	
Progression to Algebra http://ime.math.arizona.edu/progressions/	

(m)Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

(S)Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
Mathematical relationships can be presented graphically, in tables, or in verbal descriptions and the meaning of features in each representation can be interpreted in terms of the situation.	How does each element of the domain correspond to exactly one element of the range?	completing the square domain exponential function extreme values factoring function intercepts interval irrational number linear function maxima maximum minima minimum parameter range rational number relative maximum relative minimum root quantitative relationship quadratic function symmetry zeros
Quadratic, linear or exponential function can be modeled, and the situation can be used in context to specify the domain and range as it relates to the understanding of real-world application of algebra concepts.	How would you relate and interpret features of relationships represented in a graph, table, and verbal descriptions?	
The connection between the graph of the equation $y = (x)$ and the function itself can be made, and the coordinates of any point on the graph represent an input and output, expressed as $(x, (x))$.	How can you represent the same function algebraically in different forms and interpret these differences in terms of the graph or context?	
Translation between the tabular, graphical, and symbolic representations of a function can be explored between these representations and the situation's context.	What differences are there in the parameters of linear, exponential, and quadratic expressions?	
Key characteristics of functions can be identified, and function language and notation to analyze and compare functions can be used.	How would you model physical problems with linear, exponential and quadratic functions and what role would their parameters play in modeling?	
The zeros and roots of a quadratic function can be solved by factoring or completing the square.	How can you find the zeros and roots of a quadratic function? How do the graphs of mathematical models and data help us better understand the world in which we live?	
Equivalent forms of linear, exponential and quadratic functions can be created to analyze and compare functions and features of functions (e.g. rates of change in specified intervals).	How would you explain the product or sum of rational and irrational numbers?	
The same function can be represented algebraically in different forms and the differences can be interpreted in terms of the graph or context.		
"The sum or product of two rational numbers is rational" can be explained, by arguing that the sum of two fractions with a integer numerator and denominator is also a fraction of the same type.		

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs • Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Algebra I • College Preparatory Mathematics, 2013: Core Connections, Algebra I • The College Board, 2014:Springboard Algebra I **Mathematics Assessment Project – MARS Task** Function and Everyday Situations - F.IF.7-8 **Illustrative Mathematics** • Influenza Epidemic – F.IF.4 • Warming and Cooling – F.IF.4: • How is the weather – F.IF.4: • Logistic Growth Model, Explicit Version – F.IF.4 • The Canoe Trip, Variation 1 – F.IF.4-5 • The High School Gym – F.IF.6b • Temperature Change –F.IF.6 • Which Function? - F.IF.8a • Throwing Baseballs – F.IF.9 and F.IF.4	Facilitate a discussion with students that would help them represent functions with graphs and identify key features in the graph. Create or use already created activity where students would match different functions with their graphs, tables, and description. Engage students in graphing linear, exponential, and quadratic functions in order for them to develop fluency and the ability to graph them by hand. Help students to develop their idea of modeling physical problems with linear, exponential, and quadratic functions by looking at practical application of linear, quadratic, and exponential situations; such as stock market and investment, compound and simple interests, rocket trajectory, and speed of cars. Provide students the opportunity to compare linear, quadratic, and exponential functions, represented in different ways (table, graph, or situation) in writing using graphic organizers; such as T-chart or Venn diagram.	**Formative Assessment** PARCC Sample Assessments: http://www.parcconline.org/samples/mathematics/high-school-mathematics **LAUSD Assessments** The district will be using the SMARTER Balanced Interim Assessments. Teachers would use the Interim Assessment Blocks (IAB) to monitor the progress of students. Each IAB can be given twice to show growth over time. **State Assessments** California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: http://www.smarterbalanced.org/

LANGUAGE GOALS for **low achieving, high achieving, students with disabilities and English Language Learners**
Students will relate and interpret orally and in writing using complex sentences the meaning and features of relationships arising from a situation – whether presented graphically, in tabular form, and/or as verbal descriptions. Students will explain (orally and in writing) how to model a situation with a quadratic, linear or exponential function, and will be able to use the situation's context to specify the domain and range. Students will write how to translate between the tabular, graphical, and symbolic representations of a function, and between these representations and the situation's context. Students will identify and orally explain key characteristics of functions using the function language and notation to analyze and compare functions.
PERFORMANCE TASKS
Mathematics Assessment Project – MARS Task • Functions and Everyday Situations – F.IF.4- 9, F.BF.3, F.LE.3:

Illumination Mathematics

- Average Cost – F.IF.B.4-5

Noyce Foundation – Inside Mathematics

- Sorting Functions – F.IF.4, 7a, 7c, 8a, F.LE.2

DIFFERENTIATION

UDL/FRONT LOADING	ACCELERATION	INTERVENTION
Prerequisites: Understanding and use the formal mathematical language of functions. Provide students an opportunity to compare two functions (quadratic and exponential), represented in different ways (table, graph, or situation).	Provide the students several opportunities to collect data to model different situations related to linear, quadratic, exponential functions, and trigonometric functions. Have students complete a project such as: The half-life of caffeine is 6 hours. In other words, after consuming some caffeine, half of that caffeine is still present in the body after 6 hours. The amount of caffeine in the body at the end of any given time interval is A=Pd-kt where P is the amount of caffeine present in the body at the beginning of the time interval, t is the length of the time interval, and k is the decay constant. For one day from the time you wake up to the time you go to bed, keep a record of the time and the amount consumed of any beverage that contains caffeine. Research how much caffeine is in each type of drink you consumed. Calculate the amount of caffeine in your body when you went to bed that night. Compare your results with your classmates. Use your calculations and the results of others to make a conjecture about the time of day you should consume your last caffeinated beverage if you want to have less than 20 milligrams in your body when you go to sleep. What time should you consume your last caffeinated beverage if you want to have no caffeine in your body when you go to sleep? (CORD Algebra 2: Learning in Context, 2008.) Which Function? - F.IF.8a This activity is a nice analysis that involves a real understanding of what the equation of a translated parabola looks like.	Have students evaluate different functions (linear, quadratics, and exponential) for a given variable. Then engage the students in identifying appropriate domain for the functions. Help students take the "function machine" that they learned in the earlier grades and turn it into a deeper understanding of relating the situation, table, and rule (formula) of functions. The goal here is to help students make the connections.

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.
3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://www.engageny.org/resource/high-school-algebra-i.
4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
6. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.